Creep stress redistribution of FGMEE rotating disc in hygrothermal environmental condition

Mahdi Saadatfar , Yousef Iravani , H. Al-Taee , A.N.D. Al-Badr , A.S.N. Al-samarmad.

1. INTRODUCTION

- Rotating discs are subject of many researches due to their huge application in rotating machinery such as steam and gas turbine rotors, turbo generators, compressors, flywheels, automotive braking systems, ship propellers and computer disc drives.
- Several authors have analyzed the creep behavior in composite, FGM and smart discs. For composite and FGM discs:

Authors	Title
Singh and Ray	the steady-state creep behavior of a rotating disc made of Al-SiC composite
Gupta et al	creep response of an isotropic FGM rotating disc with radial thermal gradient
Rattan et al.	the creep behavior of an isotropic rotating disc made of particle reinforced FGM
Thakur et al.	the creep progress in a disc with shaft having variable density in thermal environment

2. BASIC FORMULATIONS OF THE PROBLEM

- uniform thickness FGMEE disc rotates about z axis with a constant angular velocity
- cylindrical coordinate system (r, θ, z) is considered.
- The inner and outer radius are assumed to be a and b, respectively.
- The disc is considered to be subjected to hygrothermal field as well as electro-magnetic potentials at inner and outer surfaces.
- symmetry and plane stress condition
- all material constants, are supposed to have the power-law distributions through the radial direction as $\xi(r)=\bar{\xi}r^{\beta}$,

2-1. Formulation of the hygrothermal field

- The axisymmetric steady-state Fourier heat conduction and Fickian moisture diffusion equations without source of heat and moisture for a disc are expressed as:

$$\frac{1}{r}\frac{\partial}{\partial r}(rk^{T}\frac{\partial T}{\partial r})=0,\qquad \frac{1}{r}\frac{\partial}{\partial r}(rk^{C}\frac{\partial M}{\partial r})=0,$$

$$T(r)=W_1 r^{-\beta}+W_2,\qquad M(r)=S_1 r^{-\beta}+S_2.$$

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2.2 Basic equations of the FGMEE disc

the stress-strain relations can be written in the following form: These boundary conditions can be expressed as:

$$\sigma_r=c'_{11}\partial u/\partial r+c'_{12}u/r+e'_{11}\partial\phi/\partial r+q'_{11}\partial\psi/\partial r-\lambda'_1 T-\zeta'_1 M-c'_{11}\varepsilon^c_{rr}-c'_{12}\varepsilon^c_{\theta\theta}$$
$$\sigma_\theta=c'_{12}\partial u/\partial r+c'_{22}u/r+e'_{12}\partial\phi/\partial r+q'_{12}\partial\psi/\partial r-\lambda'_2 T-\zeta'_2 M-c'_{12}\varepsilon^c_{rr}-c'_{22}\varepsilon^c_{\theta\theta}$$
$$B_r=q'_{11}\partial u/\partial r+q'_{12}u/r-\varepsilon'_{11}\partial\phi/\partial r-d'_{11}\partial\psi/\partial r+m'_1 T+\gamma'_1 M-q'_{11}\varepsilon^c_{rr}-q'_{12}\varepsilon^c_{\theta\theta}$$
$$D_r=e'_{11}\partial u/\partial r+e'_{12}u/r-\beta'_{11}\partial\phi/\partial r-\varepsilon'_{11}\partial\psi/\partial r+p'_1 T+\chi'_1 M-e'_{11}\varepsilon^c_{rr}-e'_{12}\varepsilon^c_{\theta\theta},$$

boundary conditions can be expressed as:

$$u_r(r=a)=0,\quad \sigma_r(r=b)=0,\quad \phi(r=a)=\phi_a,$$
$$\phi(r=b)=\phi_b,\quad \psi(r=a)=\psi_a,\quad \psi(r=b)=\psi_b.$$

3. SOLUTION OF THE EQUATIONS

To find initial stresses, by ignoring creep strains:

$$u=B_1r^{m_1}+B_2r^{m_2}+B_3r+B_4r^{\beta+1}+B_5r^{1-\beta}+B_6r^3+B_7A_2r^{-\beta}+B_8A_1r^{-\beta}$$

Well-known Prandtl-Reuss equations relate the creep rates to the stresses and the material creep constitutive model as follows:

$$\dot{\varepsilon}^c_r=\dot{\varepsilon}^c_e(\sigma_r-0.5(\sigma_\theta+\sigma_z))/\sigma_e$$
$$\dot{\varepsilon}^c_\theta=\dot{\varepsilon}^c_e(\sigma_\theta-0.5(\sigma_r+\sigma_z))/\sigma_e$$
$$\dot{\varepsilon}^c_z=\dot{\varepsilon}^c_e(\sigma_z-0.5(\sigma_\theta+\sigma_r))/\sigma_e$$

rates of stresses, electric potential and magnetic potential can be :

$$\dot{\sigma}_r=D_1(m_1C_1+C_2)r^{\beta+m_1-1}+D_2(m_2C_1+C_2)r^{\beta+m_2-1}+(B_8(C_2-C_1\beta)+C_4)r^{-1}\dot{A}_1-(B_7(C_2-C_1\beta)+C_3)r^{-1}\dot{A}_2+C_1r^{\beta}(G'_{11}r^{m_1}+G_{11}m_1r^{m_1-1}$$
$$+G'_{21}r^{m_2}+G_{21}m_2r^{m_2-1})+C_2r^{\beta-1}(G_{11}r^{m_1}+G_{21}r^{m_2})-b_0r^{b_1+\beta}(C_1(\sigma_r-5\sigma_\theta)-C_2(\sigma_\theta-5\sigma_r))$$

$$\dot{\sigma}_\theta=D_1(m_1E_1+2E_2)r^{\beta+m_1-1}+D_2(m_2E_1+E_2)r^{\beta+m_2-1}+(B_8(E_2-E_1\beta)+E_4)r^{-1}\dot{A}_1-(B_7(E_2-E_1\beta)+E_3)r^{-1}\dot{A}_2+E_1r^{\beta}(G'_{11}r^{m_1}+G_{11}m_1r^{m_1-1}$$
$$+G'_{21}r^{m_2}+G_{21}m_2r^{m_2-1})+E_2r^{\beta-1}(G_{11}r^{m_1}+G_{21}r^{m_2})-b_0r^{b_1+\beta}(E_1(\sigma_r-5\sigma_\theta)+E_2(\sigma_\theta-5\sigma_r))$$

$$\dot{\psi}=D_1r^{m_1}(P_1+P_2m_1^{-1})+D_2r^{m_2}(P_1+P_2m_2^{-1})-(B_8(P_2-P_1\beta)+P_3)r^{-\beta}(\beta)^{-1}\dot{A}_1-(B_7(P_2-P_1\beta)-P_5)r^{-\beta 1}(\beta)^{-1}\dot{A}_2+\int[P_1(G'_{11}r^{m_1}+G_{11}m_1r^{m_1-1}$$
$$+G'_{21}r^{m_2}+G_{21}m_2r^{m_2-1})+P_2(G_{11}r^{m_1-1}+G_{21}r^{m_2-1})+(P_1\dot{\varepsilon}^c_{rr}-P_2\dot{\varepsilon}^c_{\theta\theta})]dr+J_2$$

$$\dot{\phi}=D_1r^{m_1}(L_1+L_2m_1^{-1})+D_2r^{m_2}(L_1+L_2m_2^{-1})-(B_8(L_2-\beta L_1)-L_5)r^{-\beta}(\beta)^{-1}\dot{A}_1-(B_7(L_2-L_1\beta)+L_3)r^{-\beta}(\beta)^{-1}\dot{A}_2+\int[L_1(r^{m_1}\partial G_{11}/\partial r+G_{11}m_1r^{m_1-1}$$
$$+r^{m_2}\partial G_{21}/\partial r+G_{21}m_2r^{m_2-1})+L_2(G_{11}r^{m_1-1}+G_{21}r^{m_2-1})-L_1\dot{\varepsilon}^c_{rr}-L_2\dot{\varepsilon}^c_{\theta\theta}]dr+J_1$$

Material constants to be used can be found in Ref. [16, 19]. The inside and outside radius of the disc is taken as a=0.1m and b=0.2m, respectively.

$$R=(r-a)/(b-a),\ u^*=u/a,\ \sigma^*_i=\sigma_i/P_a,(i=r,\theta),\ \phi^*=(\bar{\beta}_{11}/\bar{c}_{11})^{1/2}(\phi/b),\ \psi^*=(\bar{d}_{11}/\bar{c}_{11})^{1/2}(\psi/b)$$

$$\phi_a=0,\quad \phi_b=5000,\quad \psi_a=0,\quad \psi_b=0,\quad T_a=100,\quad T_b=0,\quad M_a=2,\quad M_b=0$$

4. NUMERICAL RESULTS AND DISCUSSIONS

Figure 1 - Validation of the results

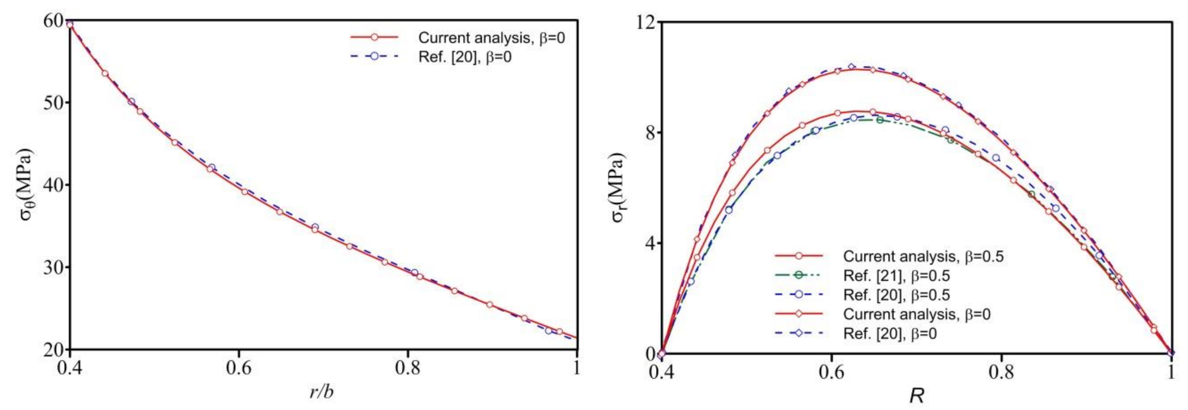


Figure 2 - (a) Radial, (b) hoop and (c) equivalent creep stresses, (d) radial displacement redistribution during creep evolution

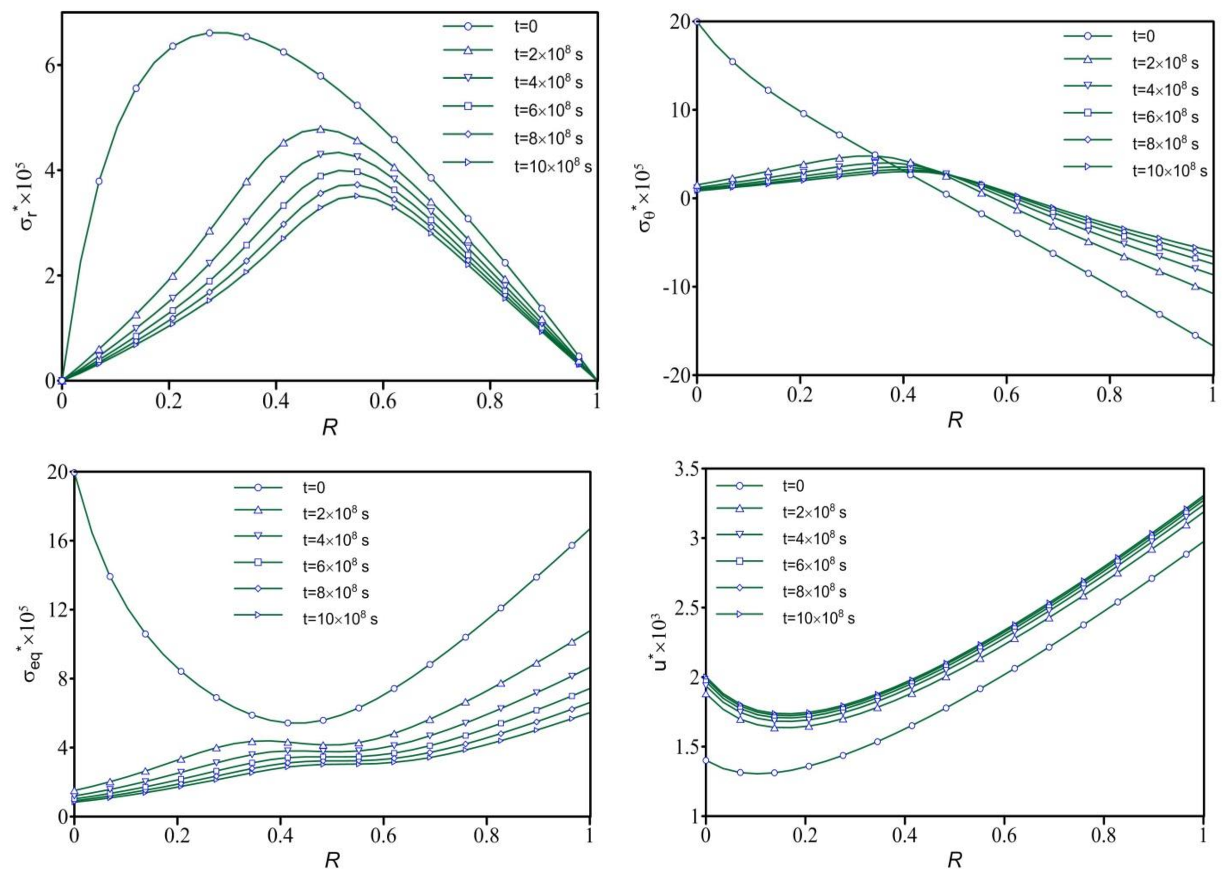
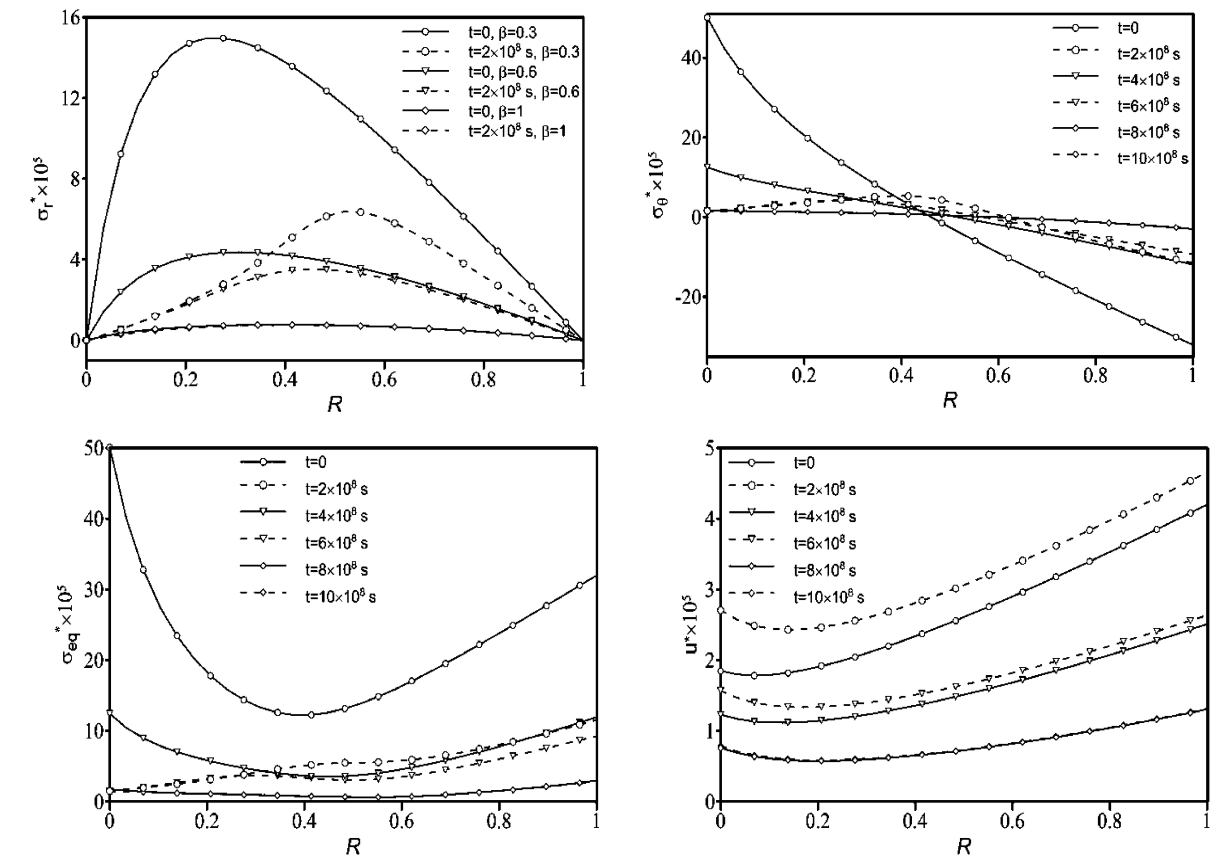


Figure 3 - Effect of grading index on the initial and creep (a) Radial, (b) hoop and (c) equivalent creep stresses, (d) radial displacement redistributions



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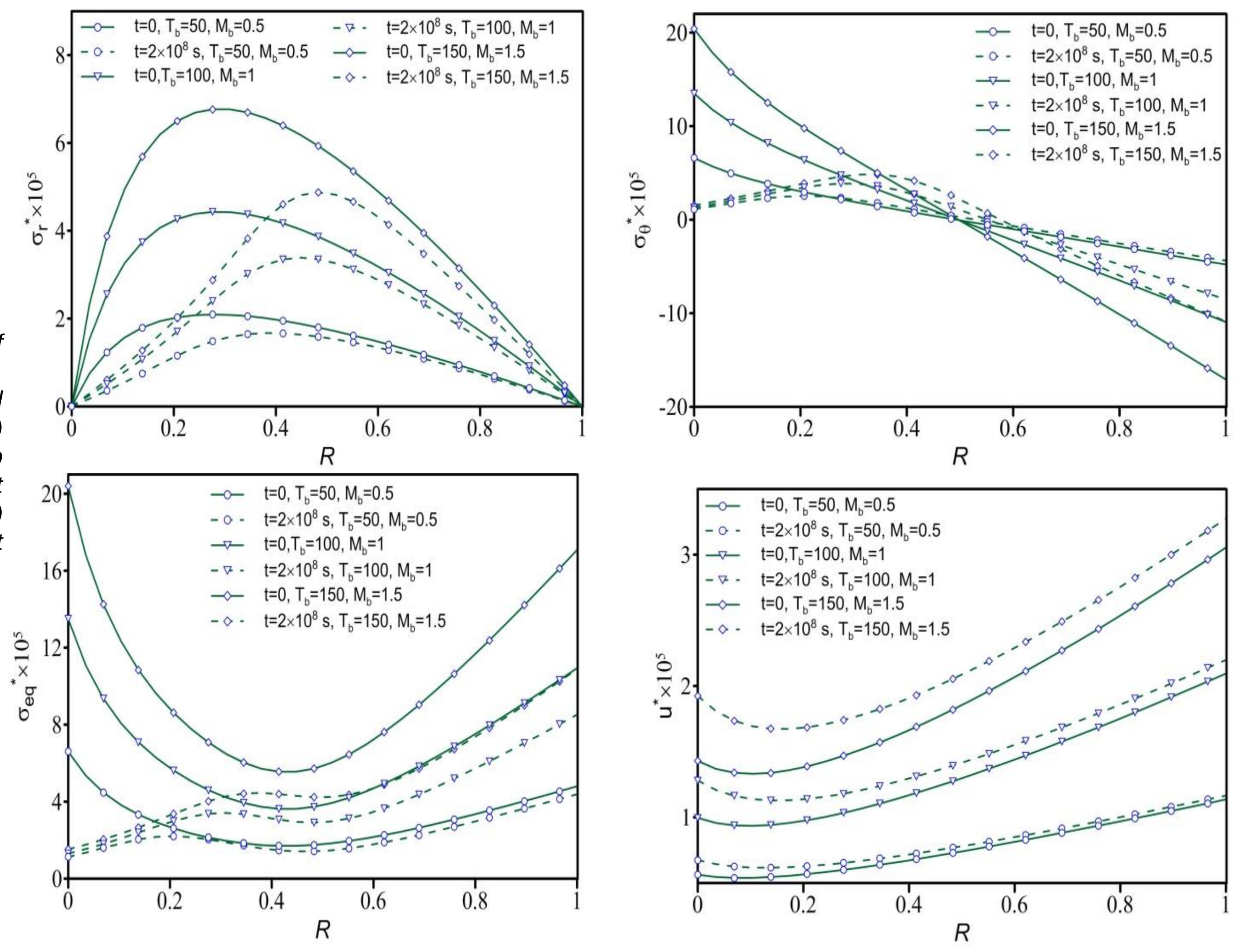

Figure 4 - Effect of hygrothermal loading on the initial and creep (a) Radial, (b) hoop and (c) equivalent creep stresses, (d) radial displacement redistributions

5. CONCLUSION

A functionally graded magneto-electro-elastic disc subjected to an axisymmetric multiphysical loading is considered. The material constants are assumed to be power-law function of radius. Applying the Prandtl-Reuss equations and Norton's law, the time-dependent creep behavior of the disc is analyzed. The following conclusions are found in the analysis:

- By serving the time, the radial stress rises, the absolute value of hoop stress decreases, the equivalent stress decreases, and the outward radial displacement rises with decreasing rate.
- Rising in grading index leads to a reduction in the initial and creep radial stress together with a reduction in both the initial hoop stress and initial equivalent stress. Also, it leads to a reduction in the radial displacement for both initial and creep states.
- Rising in hygrothermal loading rises the radial stress, equivalent stress, and radial displacement both for initial and creep cases.

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Thank you very much
For your attention

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